Added Resistance of Ships in Waves

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A new method is presented for computing the added resistance of a ship advancing at constant mean speed and heading in regular waves. It is shown that the added resistance, even though it is a second-order quantity, can be expressed as a product of first-order terms that are all computed by computer programs presently in use for predicting the linear heave and pitch motions. Comparisons between the present theory and other theories, as well as experimental results, are presented for head waves. It is found that the discrepancies in the experimental results can be as large as the differences between different theories. A difference of 30% between experimental values for the maximum added resistances seems to be typical. It is concluded that the present theory predicts the added resistance with better accuracy for a wider range of ships forms and speeds than any other existing theory. It is also shown how the mean added resistance in a short-crested seaway can easily be obtained from the regular-wave results.

Nomenclature

A_{jk} = added-mass coefficients
B = ship beam
B_{jk} = damping coefficients
C_B = block coefficient
F_n = Froude number, $F_n = U/\sqrt{gL}$
F_j = exciting force and moment
F_j^I = Froude-Krilov exciting force and moment
F_j^D = diffraction exciting force and moment
L = length between perpendiculars
N = two-dimensional outward unit normal vector
R = added resistance
S = hull surface at mean position
U = ship speed
a_{jk} = two-dimensional sectional added-mass coefficient
b_{jk} = two-dimensional section damping coefficient
g = gravitational acceleration
k = wave number
n = three-dimensional outward unit normal vector
t = time variable
x,y,z = coordinate system fixed with respect to mean position of ship
α = incident wave amplitude
β = angle between mean wave direction and ship heading ($\beta = 180$ deg for head seas)
ζ_j = displacements
ρ = mass density of water
σ = wave frequency
ϕ_0 = incident wave potential
ϕ_B = body-disturbance potential
ϕ_j = potential due to jth mode of motion
ϕ_7 = diffraction potential
ψ_j = two-dimensional potential
ω = frequency of encounter

I. Introduction

THIS paper presents a new method of computing the added resistance of a ship advancing at a constant mean speed and heading in a seaway. This method is shown to be applicable over a much wider range of ship forms and speeds than any other existing technique.

Although the added resistance of a ship in a seaway is a highly nonlinear phenomenon, it has been found that the principle of superposition, as first used in seakeeping by St. Denis and Pierson,[1] is applicable to this problem. This assumption permits the calculation of the added resistance in a seaway to be reduced to two problems: a) the prediction of the added resistance in regular waves and b) the prediction of the statistical mean values in irregular waves using the regular wave results. These two aspects of the added-resistance problem will be discussed separately.

A. Added Resistance in Regular Waves

There exist several methods for determining the added resistance in regular waves; among these the methods of Havelock,[2] Maruo,[3] Joosen,[4] and Gerritsma and Beukelman[5] are the most generally known. Unfortunately, as shown by Strom-Tejsen et al.[6] in an evaluation of these methods, none of them predict the added resistance accurately over a wide range of ship forms and speeds. As is generally known and is stated in Ref. 6, the theory of Havelock[2] must be considered a first-order approximation that is not accurate enough for most engineering applications. It was found in Ref. 6 that Maruo's theory[3] gives accurate results only for cruiser-stern ships without large bulbous bows and that it is not applicable to other hull forms. Maruo states, in his discussion of Ref. 6, that in his theory "the ship hull is represented by a line distribution of sources" and that "since the density of the source distribution is proportional to the width of the load waterline, it cannot take into account the effect of the bulbous bow." Furthermore, the poor agreement between his theory and experiments as shown in Ref. 6 for the high-speed destroyer form seems to demonstrate that his theory is not generally applicable to transom-stern ships. It was found in Ref. 6 that the theory of Joosen[4] gives fair results for transom stern ships, but is less accurate for cruiser-stern ships. Strom-Tejsen et al.[6] conclude that "the method developed by Gerritsma and Beukelman[5] apparently provides a prediction technique equally accurate for all ship forms." A careful examination of their results does not seem to warrant such a conclusion. Figure 1 is a reproduction of Fig. 8a in Ref. 6 and clearly demonstrates that for the case of a Series 60 ship with $C_B = 0.60$, the theory predicts maximum added resistances that are more than twice as large as those measured. The analytic results shown in Fig. 1 were computed by the method of Gerritsma and Beukelman using ship motions as predicted by the theory of Salvesen et al.[7]

Consequently, I believe that we can conclude from the investigation conducted by Strom-Tejsen et al.[6] that the added-resistance method of Gerritsma and Beukelman, when used with the motion theory of Salvesen et al.,[7] has a wider range of application than the other theories investigated in

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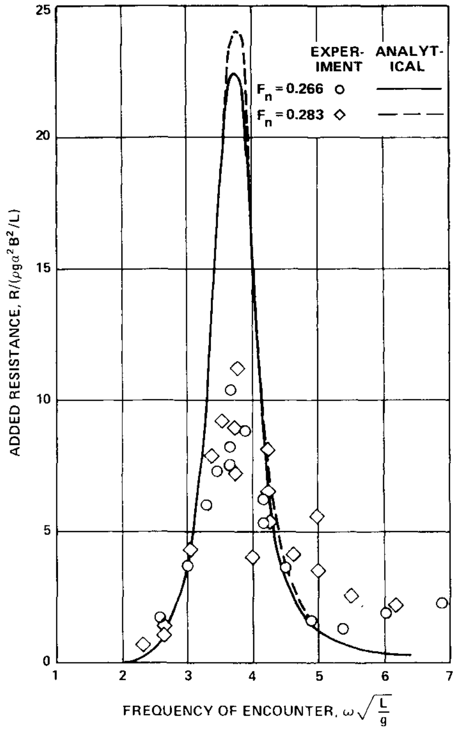

Fig. 1 Comparison of experimental and theoretical added resistance for Series 60 hull with $C_B = 0.60$ (from Ref. 6).

Ref. 6; however, as seen in Fig. 1, this approach does not seem to predict added resistances that agree well with experiments for cruiser-stern ships with low block coefficients. It should be recognized that we cannot expect the same degree of agreement between theory and experiments for the added resistance as we have become accustomed to in heave and pitch motions. The added resistance is a nonlinear quantity where, according to the conventional formulation of the ship-motion problem, the leading term is of second-order in the wave amplitude. This means that if the linear motion responses are predicted with an accuracy of approximately 10-15%, we cannot expect the second-order added resistance to be of accuracy better than approximately 20-30%.

It is important to note that all of the computational added-resistance prediction methods use the heave and pitch motion responses as input values and that the added resistance predicted by a given method may vary considerably depending on the method used for obtaining the motions. For example, better agreement was found in Ref. 6 for the case shown in Fig. 1 when experimentally obtained ship-motion responses were used together with the added resistance theory of Gerritsma and Beukelman; however, this fact is of less practical interest since we are mainly interested in a strictly computational tool for predicting the added resistance. Gerritsma and Beukelman[12] have shown that if their added-resistance method is used together with their own technique for predicting the ship motions, the agreement between computed and experimental results is in many cases better than when their added-resistance method is used with motions obtained by the method of Salvesen et al.[7] In this paper, as well as in Strom-Tejsen et al.,[6] all of the computed motions have been obtained by the method of Salvesen et al.; however, it is recognized that the final added-resistance results could be different, and even in some cases closer to the experimental data, if other techniques were used for predicting the motions.

Salvesen[8] has developed a theory for predicting the second-order mean forces and moments on a ship in oblique regular waves. There are some assumptions applied in the derivation of this theory that make some of the final results questionable. However, if this theory is only used to predict the force component in the longitudinal direction which is the added resistance, none of these assumptions affect the derivation. In fact, it is shown here that the added-resistance problem can be formulated subjected to the same assumption used in deriving the ship-motion strip theory of Salvesen et al.[7] Furthermore, it is shown that the added resistance in oblique waves, even though it is a second-order quantity, can be expressed as a product of first-order terms that are all computed by the computer programs presently in use for predicting the linear heave and pitch responses in oblique waves. Comparisons between theories and experiments are presented for head waves; these seem to demonstrate that the present theory has a wider range of application than the theory of Gerritsma and Beukelman.

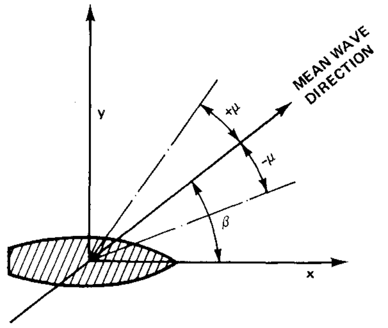

Fig. 2 Definitions of heading β and spreading μ angles.

B. Added Resistance in Irregular Waves

Consider that the added resistance in regular waves $R(U,\beta,\sigma)$ is a known function of the ship's mean forward speed U, the heading angle β, and the wave frequency σ. (The heading angle is defined in Fig. 2.) Furthermore, if it is assumed that the principle of superposition applies to the added-resistance problem, and that the irregular sea can be represented by a two-dimensional energy spectrum $S(\mu,\sigma)$, where μ is the spreading angle, then the mean added resistance in a seaway for a ship advancing at a given speed and heading is

$$\bar{R}(U,\beta) = 2\int_0^\infty \int_{-\pi/2}^{\pi/2} R(U,\beta+\mu,\sigma)S(\mu,\sigma)\,d\mu d\sigma \quad (1)$$

Following the recommendations made by the 12th International Towing Tank Conference in Rome in 1969, we may express the sea energy spectrum as

$$S(\mu,\sigma) = K(\mu)S(\sigma) \quad (2)$$

where the recommended spreading function is

$$K(\mu) = \frac{2}{\pi}\cos^2\mu \quad (3)$$

and where the recommended and most commonly used one-dimensional spectrum is the Pierson-Moskowitz spectrum.

$$S(\sigma) = \frac{A}{\sigma^5}e^{-B/\sigma^4} \quad (4)$$

The coefficients in the spectrum (4) can be expressed in terms of two parameters, the significant wave height $h_{1/3}$ and the characteristic period T, as

$$A = 173h_{1/3}^2/T^4 \text{ and } B = 691/T^4 \quad (5)$$

or they may be expressed in terms of only one parameter, the significant wave height, as

$$A = 0.0081g^2 \text{ and } B = 33.56/h_{1/3}^2 \quad (6)$$

Here g is the gravitational acceleration and $H_{1/3}$ is in feet. The Pierson-Moskowitz spectrum (4) is widely used both as a one-parameter spectrum (6) or as a two-parameter spectrum (5); however, it should be noted that the one-parameter spectrum is only adequate in representing full developed seaways, whereas the two-parameter spectrum may be used to represent developing seas and swells as well.

It may be sufficiently accurate for many design applications to disregard the spreading function (3) and simply represent the seaway by an unidirectional spectrum (4). The mean added resistance is then given by

$$\bar{R}(U,\beta)=2\int_0^\infty R(U,\beta,\sigma)S(\sigma)\,\mathrm{d}\sigma \tag{7}$$

There exists no experimental verification for the general case of the mean added resistance in an oblique seaway (1); however, several investigations have indicated that the principle of superposition applies to the case of unidirectional head waves. For example, Nakamura[16] has recently found quite acceptable agreement between experimental values of mean added resistance in irregular head waves and values obtained by Eq. (7) using experimental regular-wave responses R and the actual spectrum S, measuring during the irregular-wave experiments.

In other words, if it is assumed that the principle of superposition applies to the added-resistance problem, it is a simple computational procedure to obtain the mean added resistance in an irregular seaway, if the added resistance in regular waves $R(U,\beta,\sigma)$ is known. Thus, the main objective here will be to present and to evaluate a new method for computing the added resistance in regular waves.

II. Linear Ship-Motion Strip Theory

This section gives a brief review of the linear strip theory for ship motions. The work of Salvesen et al.[7] is followed closely.

Consider a ship advancing at constant mean forward speed U with arbitrary heading in regular sinusoidal waves. It is assumed that the resulting oscillatory motions are linear and harmonic. Let (x,y,z) be a right-handed orthogonal coordinate system fixed with respect to the mean position of the ship, with z vertically upward through the ship's center of gravity, x in the direction of forward motion, and the origin in the plane of the undisturbed free surface. Suppose that the ship oscillates as a rigid body in 6 degrees of freedom with complex amplitudes ζ_j ($j=1, 2. . . .6$). Here $j=1, 2, 3, 4, 5,$ and 6 refer to surge, sway, heave, roll, pitch, and yaw motions, respectively.

If viscous effects are disregarded, the fluid motions can be assumed to be irrotational so that the problem can be formulated within potential flow theory. The velocity potential for an incident wave is, according to linear gravity-wave theory,

$$\Phi_0(x,y,z;t)=\phi_0(x,y,z)e^{i\omega t} \tag{8}$$

where the complex amplitude of the incident-wave potential is[†]

$$\phi_0=\frac{ig\alpha}{\sigma}\exp[-ik(x\cos\beta+y\sin\beta)+kz] \tag{9}$$

Here α is the wave amplitude, k the wave number, β the heading angle ($\beta=0$ for following waves), and σ the wave frequency which is related to the frequency of encounter ω through

$$\sigma=\omega+kU\cos\beta \tag{10}$$

[†]It is to be understood that the real part is to be taken in expressions involving $e^{i\omega t}$.

If it is assumed that the ship has lateral symmetry (symmetry about the x,z plane), it can be shown that the surge, heave, and pitch motions are decoupled from the sway, roll, and yaw motions. Furthermore, if it is also assumed that the ship has a long slender hull form, it is consistent with the assumptions of strip-theory to neglect surge. Hence, for a slender ship with lateral symmetry, the two linear coupled equations that govern the heave and pitch motions in the frequency domain are

$$\sum_{k=3,5}\left[-\omega^2(M_{jk}+A_{jk})+i\omega B_{jk}+C_{jk}\right]\zeta_k$$
$$=F_j \text{ for } j=3 \text{ and } 5 \tag{11}$$

where M_{jk} is the generalized mass matrix for the ship, A_{jk} and B_{jk} are the added-mass and damping coefficients, C_{jk} are the hydrostatic restoring coefficients, and F_j are the complex amplitudes of the exiciting force and moment. We will not consider the sway, yaw, and roll motions in this work, since it has been shown in Ref. 8 that these motions have a very small contribution to the added resistance.

It is shown in Ref. 7 that the speed- and frequency-dependent hydrodynamic coefficients, the A_{jk} and B_{jk} of Eq. (11), can be expressed in terms of the two-dimensional sectional added-mass and damping coefficients, a_{jk} and b_{jk}, as follows

$$A_{33}=\int a_{33}\mathrm{d}x \tag{12}$$

$$B_{33}=\int b_{33}\mathrm{d}x \tag{13}$$

$$A_{35}=-\int xa_{33}\mathrm{d}x-\frac{U}{\omega^2}B_{33} \tag{14}$$

$$A_{53}=-\int xa_{33}\mathrm{d}x+\frac{U}{\omega^2}B_{33} \tag{15}$$

$$B_{35}=-\int xb_{33}\mathrm{d}x+UA_{33} \tag{16}$$

$$B_{53}=-\int xb_{33}\mathrm{d}x-UA_{33} \tag{17}$$

$$A_{55}=\int x^2a_{33}\mathrm{d}x+\frac{U^2}{\omega^2}A_{33} \tag{18}$$

$$B_{55}=\int x^2b_{33}\mathrm{d}x+\frac{U^2}{\omega^2}B_{33} \tag{19}$$

where the integrations are over the length of the ship. The added-mass and damping coefficients, A_{jk} and B_{jk}, given in Ref. 7 have some additional end terms not included in Eqs. (12)-(19). Recent unpublished work seems to indicate that the agreement between computed and experimental data is generally not improved by including these end terms in the coefficients. Hence, they have not been included here, or in any of the computations used in this paper.[‡]

The fact that it is possible to express the hydrodynamic coefficients A_{jk} and B_{jk} in terms of the sectional coefficients a_{jk} and b_{jk} is the basis of the strip theory, since it means that the only hydrodynamic problems that have to be solved to predict A_{jk} and B_{jk} are two-dimensional potential-flow problems of cylinders heaving in the free surface. This

[‡]Note that in the heave and pitch ship-motion computer program of Frank and Salvesen[9] the pitch damping coefficient B_{55} does not include the speed term U^2B_{33}/ω^2 given in Eq. (19). The absence of this term can result in considerable discrepancy for high-speed ships.

problem may be solved by representing the sections by Lewis forms, as done by Grim,[10] or by using the close-fit source distribution method first used by Frank.[11]

The exiciting force and moment may be expressed as the following sum

$$F_j = F_j^I + F_j^D \tag{20}$$

where the first term in the sum is the Froude-Krilov force and moment

$$F_3^I = -i\rho\sigma \int\int_s N_3 \phi_0 \mathrm{d}s \tag{21}$$

and

$$F_5^I = i\rho\sigma \int\int_s x N_3 \phi_0 \mathrm{d}s \tag{22}$$

Here the integrations are over the mean position of the hull surface S. The last term in Eq. (20) is the diffraction force and moment

$$F_3^D = \int_L h_3(x)\mathrm{d}x \tag{23}$$

and

$$F_5^D = -\int_L \left(x + \frac{U}{i\omega}\right) h_3(x)\mathrm{d}x \tag{24}$$

with

$$h_3(x) = \rho k \int_C \psi_3 (N_3 + iN_2 \sin\beta)\phi_0 \mathrm{d}l \tag{25}$$

Here ρ is the mass density of the fluid, N_2 and N_3 are the components in the y and z directions of the two-dimensional outward unit normal vector in the y,z plane, and dl is an element of arc along the cross section C. Furthermore, ψ_3 is the velocity potential for the two-dimensional problem of a cylinder oscillating in heave in the free surface. Hence, the exciting force and moment F_j as well as the added-mass and damping coefficients A_{jk} and B_{jk} can all be expressed in terms of the solution of the two-dimensional problem of a cylinder oscillating in the free surface.

Since the assumptions used in the derivation of the ship-motion strip theory[7] are also necessary in the added-resistance theory, the most significant ones are listed:

1) The responses are assumed to be linear and harmonic.

2) The viscous effects are disregarded, so that the fluid motions can be assumed irrotational.

3) The ship is assumed slender and to have lateral symmetry.

4) The steady wave-resistance perturbation potential ϕ_s is assumed to be small so that cross-product terms between ϕ_s and the unsteady potential ϕ_T can be disregarded. This means that it is assumed that the waves generated by the ship advancing in calm water do not affect the ship motions.

5) It is assumed that the frequency of encounter is high, $\omega \gg U(\partial/\partial x)$.

In spite of this last assumption, strip theory predicts the heave and pitch motions very accurately in the low-frequency range, since in this range these motions are dominated by the hydrostatic restoring forces and these forces are not affected by this assumption.

III. Added Resistance in Oblique Waves

The derivation of the added resistance is quite similar to the derivation of the second-order mean forces and moments on surface ships in oblique waves by Salvesen.[8] Only the major steps will be given here.

It is shown in Ref. 8 that if the complex amplitude of the total potential is written as

$$\phi = \phi_0 + \phi_B \tag{26}$$

where ϕ_0 is the incident wave potential and ϕ_B is the potential due to the body disturbance, including diffraction effects, the second-order mean steady force can be written as

$$\mathfrak{F} = -\frac{1}{2}\rho \int\int_s \left(\phi_B \frac{\partial}{\partial n} - \frac{\partial \phi_B}{\partial n}\right)\left(\nabla\phi_0^* + \frac{1}{2}\nabla\phi_B^*\right)\mathrm{d}s \tag{27}$$

Here ϕ_0^* and ϕ_B^* are complex conjugates of ϕ_0 and ϕ_B. It will further be assumed that the body is a "weak scatterer" so that the body potential ϕ_B is small compared to the incident-wave potential ϕ_0. This assumption is justified if the body is slender in the sense that two of its principal dimensions are small compared with the wavelength. Ship hulls are usually slender in the sense that both beam and draft are much smaller than their length; however, this does not mean that the beam and draft are in general much smaller than the wave length. For a normal ship form, the maximum added resistance in head waves at zero forward speed occurs at wavelengths that are approximately three and a half times the beam, whereas at Froude number 0.25, it occurs when the wavelengths are about seven times the beam. Thus, it may be expected that the assumptions that $\phi_B \ll \phi_0$ will lead to accurate results for surface ships in bow and head waves at normal operating speeds, whereas at zero forward speed and in quartering and following waves, the results may be less accurate.[§]

The assumption that $\phi_B \ll \phi_0$ justifies the neglect of terms that are quadratic in ϕ_B so that the force (27) can be written as

$$\mathfrak{F} = -\frac{1}{2}\rho \int\int_s \left(\phi_B \frac{\partial}{\partial n} - \frac{\partial \phi_B}{\partial n}\right)\nabla\phi_0^* \mathrm{d}s \tag{28}$$

Since the added resistance R is equal to the negative of the x component of this force, introduction of the incident-wave potential (9) in Eq. (28) shows that the added resistance is

$$R = \frac{i}{2}\rho k \cos\beta \int\int_s \left(\phi_B \frac{\partial}{\partial n} - \frac{\partial \phi_B}{\partial n}\right)\nabla\phi_0^* \mathrm{d}s \tag{29}$$

If we write the body potential as

$$\phi_B = \sum_{j=3,5} \zeta_j \phi_j + \phi_7 \tag{30}$$

where ϕ_j is the contribution to the potential from the jth mode of motion and ϕ_7 is the diffraction potential, the added resistance (29) becomes

$$R = \frac{i}{2}\rho k\cos\beta \sum_{j=3,5} \left\{\zeta_j \int\int_s \left(\phi_j \frac{\partial}{\partial n} - \frac{\partial\phi_j}{\partial n}\right)\phi_0^* \mathrm{d}s\right\} + \frac{i}{2}\rho k \cos\beta \int\int_s \left(\phi_7 \frac{\partial}{\partial n} - \frac{\partial \phi_7}{\partial n}\right)\phi_0^* \mathrm{d}s \tag{31}$$

Now let

$$R = \sum_{j=3,5} \left\{R_j^I + R_j^D\right\} + R_7 \tag{32}$$

[§]In a recent publication, Lin and Reed[17] have included the body potential ϕ_B in their derivation of the mean force and moment. Unfortunately, they have no numerical results.

with

$$R_j^I = -\frac{i}{2}\rho k\cos\beta\zeta_j \int\int_s \frac{\partial\phi_j}{\partial n}\phi_0^* \mathrm{d}s \tag{33}$$

$$R_j^D = \frac{i}{2}\rho k\cos\beta\zeta_j \int\int_s \phi_j \frac{\partial\phi_0^*}{\partial n} \mathrm{d}s \tag{34}$$

and

$$R_7 = \frac{i}{2}\rho k\cos\beta \int\int_s \phi_7 \frac{\partial\phi_0^*}{\partial n} \mathrm{d}s \tag{35}$$

In Eqs. (33)-(35), the last term in the second integral in Eq. (31) has not been included since it can be shown by applying the hull boundary condition, $\partial\phi_7/\partial n = -\partial\phi_0/\partial n$, that this last term becomes

$$\frac{i}{2}\rho gk\cos\beta\alpha^2 \int\int_s e^{2kz}(n_3 - in_2\sin\beta)\mathrm{d}s$$

where the real part is

$$\frac{1}{2}\rho gk\cos\beta\sin\beta\alpha^2 \int\int_s e^{2kz}n_2\mathrm{d}s$$

which is identically zero for ships with lateral symmetry.

If the steps in Ref. 8 are followed, it is seen that the added-resistance component (33) can be written as

$$R_j^I = -\frac{i}{2}k\cos\beta\zeta_j(F_j^I)^* \tag{36}$$

where $(F_j^I)^*$ is the complex conjugate of the Froude-Krilov part of the exciting force and moment, F_j^I (Eqs. 21 and 22). In Ref. 8, it is shown that the second component of the added resistance (34) can be simplified to

$$R_j^D = -\frac{i}{2}k\cos\beta\zeta_j\hat{F}_j^D \tag{37}$$

where $\hat{F}_j^D$ is the same as the diffraction part of the exciting force F_j^D Eqs. (23) (25) except that in $\hat{F}_j^D$ the complex conjugate of the incident-wave potential, ϕ_0^*, appears instead of ϕ_0.

It was shown in Ref. 8 that the third added-resistance component (35) can be expressed in terms of the sectional damping coefficients b_{jk}. However, a new derivation is presented in the Appendix that gives the same result as in Ref. 8 without applying several of the assumptions used in Ref. 8. In this new derivation, the only assumptions made in addition to the strip-theory assumptions listed in the previous section are that in the integral in Eq. (35) we may replace exp(kz) by exp($-kds$) and exp(iky sinβ) by exp($\pm$ ½$ikbs$ sinβ). Here d and b are the sectional draft and beam, respectively, and s is the sectional-area coefficient. The first one of the two additional assumptions is often used in strip-theory computations (see for example Ref. 7) and has been shown to give accurate results for conventional ship-hull forms. We have less experience with the second assumption, but if the wavelength is considerably larger than the half beam, it is certainly reasonable. It is shown in the Appendix that by applying these assumptions Eq. 35 can be expressed as

$$R_7 = -\frac{1}{2}\alpha^2\frac{\sigma^2}{\omega}k\cos\beta\int_L e^{-2kds}(b_{33} + b_{22}\sin^2\beta)\mathrm{d}x \tag{38}$$

Here b_{33} and b_{22} are the sectional damping coefficents in heave and sway, respectively.

Thus, we have found that the added resistance in oblique waves is ¶

$$R = -\frac{i}{2}k\cos\beta\sum_{j=3,5}\zeta_j\left\{(F_j^I)^* + \hat{F}_j^D\right\} + R_7 \tag{39}$$

where R_7 is given by Eq. (38) and where the conjugate of the Froude-Krilov exciting force (21) and moment (22) are

$$(F_3^I)^* = i\rho\sigma\int\int_s N_3\phi_0^*\mathrm{d}s \tag{40}$$

and

$$(F_5^I)^* = -i\rho\sigma\int\int_s xN_3\phi_0^*\mathrm{d}s \tag{41}$$

and where the term $\hat{F}_j^D$ which is closely related to the diffraction force and moment F_j^D Eqs. (23)-(25) is given by

$$\hat{F}_3^D = \int_L \hat{h}_3(x)\mathrm{d}x \tag{42}$$

and

$$\hat{F}_5^D = -\int_L \left(x + \frac{iU}{\omega}\right)\hat{h}_3(x)\mathrm{d}x \tag{43}$$

with

$$\hat{h}_3(x) = -\rho k\int_C \psi_3(N_3 + iN_2\sin\beta)\phi_0^*\mathrm{d}l \tag{44}$$

Here we recall that ψ_3 is the velocity potential for the two-dimensional sectional problem of a cylinder oscillating in heave in the free surface. Examination of Eqs. (38)-(44) shows that the added resistance has been expressed in terms of the complex amplitudes of the heave and pitch motions, ζ_3 and ζ_5 and other quantities normally computed in ship-motion computer programs for oblique seas.

Figure 3, which is from Salvesen,[8] shows the added resistance predicted by the present theory for the Mariner hull form at a speed of $F_n = 0.194$ and at four different headings,

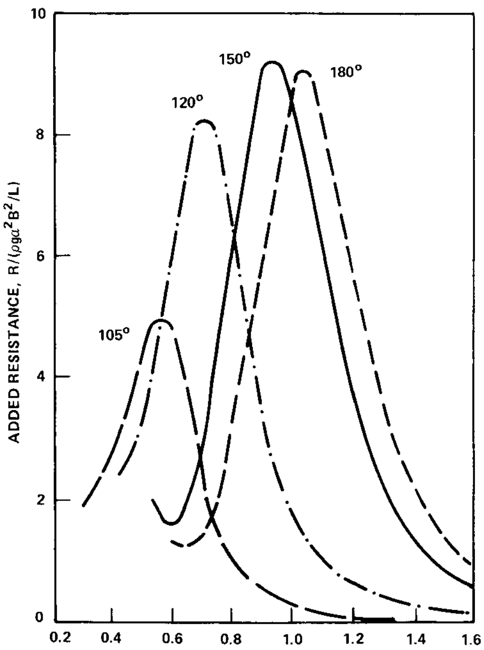

Fig. 3 Computed added resistance for mariner hull at $F_n = 0.194$ in head and bow waves.

¶It is understood that in the expressions for the added resistance, the real part is to be taken.

$\beta=105$, 120, 150, and 180 deg. (Note that $\beta=180$ deg for head waves.) As would be expected, it is seen that the maximum added resistances occur at shorter wavelengths for decreasing heading angles. It is also seen that the maximum value can be slightly larger in bow waves ($\beta=150$ deg) than in head waves. Although this result is unexpected, it does not seem to be unreasonable considering that the added resistance is the product of the exciting force, the motions and the phase angle between them, all of which are quantities which may increase or decrease with changes in heading and wavelength.

IV. Added Resistance in Head Waves

For head waves ($\beta=180$ deg) the added resistance (39) becomes

$$R=\frac{i}{2}k\sum_{j=3,5}\zeta_j\left\{(F_j^I)^*+\hat{F}_j^D\right\}+R_7 \tag{45}$$

which is more conveniently expressed as

$$R=\frac{i}{2}k\left\{\zeta_3\hat{F}_3+\zeta_5\hat{F}_5\right\}+R_7 \tag{46}$$

where

$$\hat{F}_j=(F_j^I)^*+\hat{F}_j^D \tag{47}$$

Furthermore, for head waves the complex amplitude of the incident-wave potential (9) is

$$\phi_0=\frac{i\alpha g}{\sigma}e^{ikx}e^{kz} \tag{48}$$

Introduction of Eq. (48) into the relationships (40)-(44) for the added resistance in oblique waves gives

$$\hat{F}_3=\alpha\int_L e^{-ikx}e^{-kds}\left\{\rho gb-\sigma(\omega a_{33}-ib_{33})\right\}\mathrm{d}x \tag{49}$$

and

$$\hat{F}_5=-\alpha\int_L e^{-ikx}e^{-kds}\left\{\rho gb-\sigma\left(x+\frac{iU}{\omega}\right)\left(\omega a_{33}-ib_{33}\right)\right\}\mathrm{d}x \tag{50}$$

Examination of Eq. (38) shows that for head waves

$$R_7=\frac{1}{2}\alpha^2k\frac{\sigma^2}{\omega}\int_L e^{-2kds}b_{33}\mathrm{d}x \tag{51}$$

Consequently, in head waves, the added resistance can be computed directly from the complex amplitudes of the heave and pitch motions, ζ_3 and ζ_5 and the sectional heave added-mass and damping coefficients a_{33} and b_{33}. Of course, the sectional beam b, the sectional draft d, and the sectional-area coefficent s must be known. It should be noted that the sectional beam b is the total beam and not the half-beam and that the sectional-area coefficient is defined as the sectional area divided by the sectional beam times the sectional draft.

Havelock[2] showed that to a first-order of approximation the added resistance could be expressed as

$$R=-\frac{1}{2}k\sum_{j=3,5}|\zeta_j|\,|F_j^I|\sin\delta_j \tag{52}$$

where δ_j is the phase angle between the excitation and the motion. This expression (52) is the same as the product of the motion and the conjugate of the Froude-Krilov exciting force and moment and thus is indentical to the first term in Eq. (45).

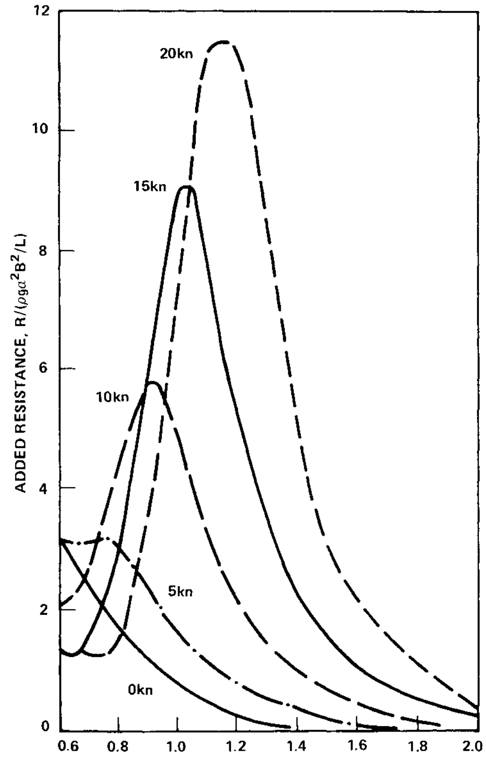

Fig. 4 Computed added resistance for mariner hull (LPB = 161 m; 528 ft) at different forward speeds in head waves.

The results in Fig. 4, which are from Salvesen,[8] and are calculated by Eq. (46) shows the large effect of the forward speed on the added resistance in head waves for a typical cruiser-stern ship. Figure 4 shows that by increasing the speed from 10 to 20 knots the maximum added resistance increases by approximately a factor of two.

V. Comparisons of Theories and Experiments

Comparisons of theories and experiments are presented here only for head waves, since there is very little data available for oblique waves. For two of the cases investigated, four different sets of experimental data are used. Comparisons are presented for three Series 60 cruiser-stern hull forms: a) a typical medium full hull form with block coefficient $C_B=0.70$ at a Froude number $F_n=0.200$, b) a fine hull form with $C_B=0.60$ at $F_n=0.283$, and c) a full hull form with $C_B=0.80$ at $F_n=0.165$. All of these speeds are close to the normal operating speeds for these block coefficients. Comparisons are also shown for two high-speed destroyer hull forms one with $C_B=0.49$ at $F_n=0.250$ and $F_n=0.350$ and the other with $C_B=0.45$ at $F_n=0.20$, 0.30, 0.40, and 0.50.

Figure 5 shows a comparison of experimental and theoretical added resistances in regular head waves for the Series 60 hull form with $C_B=0.70$ at $F_n=0.200$. Three theoretical curves are presented: Gerritsma and Beukelman's theory,[5] Maruo's theory,[3] and the present theory. Experimental data are given for four sets of experiments: experiments conducted at the David W. Taylor Naval Ship Research and Development Center with constant model speeds,[6] a the University of Technology, Delft,[12] and at the University of California, Berkeley with constant model speed[13] and constant tow force.[14]

It is seen in Fig. 5 that in the range of maximum added resistances, the experimental data differ by as much as 30%

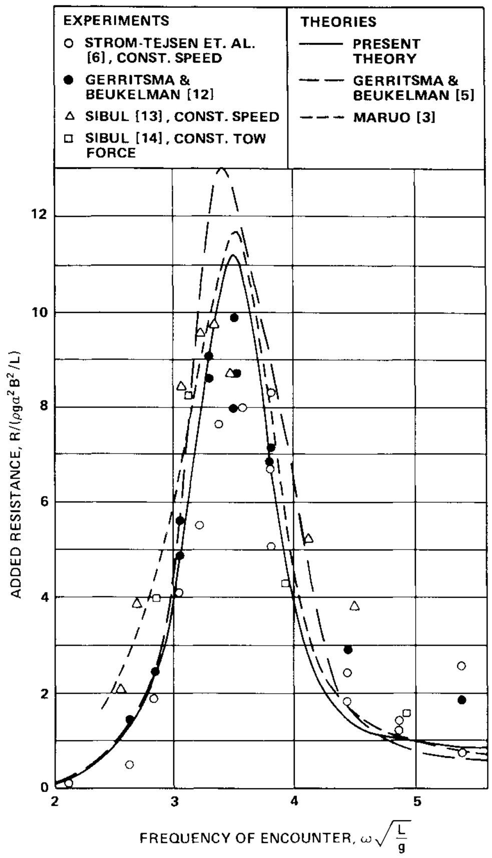

Fig. 5 Comparison of experimental and theoretical added resistance for Series 60 hull with $C_B = 0.70$ at $F_n = 0.200$.

and that in the lower and higher frequency ranges, the differences are even higher. It is extremely difficult to obtain accurate data for the added resistance since it is a small percentage of the total resistance and is determined as the difference between the resistances in waves and in calm water. In contrast, the theories are in closer agreement with each other than are the experimental data. In the range of the maximum added resistances, the differences between the theories are at the most only 16%. For this particular case, all the theories predict maximum added resistances somewhat higher than all of the experimental data.

It should be stressed that one must be very careful in judging one theory as more accurate than another from such comparisons since added resistance is a second-order quantity that is extremely sensitive to the particular ship-motion theory being used and to the computational accuracy. Computations performed by different investigators using the same added-resistance theory can differ considerably. For example, the results shown for Gerritsma and Beukelman's theory[5] in Fig. 5 are as computed by Gerritsma and Beukelman[12] using the Salvesen et al.[7] ship-motion theory, whereas Strom-Tejsen et al.,[6] using the same added-resistance and ship-motion theories, show a maximum added resistance which is about 15% higher.

Figure 6 shows added-resistance results for the Series 60 hull form with $C_B = 0.60$ at $F_n = 0.283$. Experimental data obtained by Strom-Tejsen et al.[6] for constant model speed are shown as well as a curve obtained by interpolating experimental data by Sibul[13] for $F_n = 0.250$ and $F_n = 0.350$ at both constant model speeds and constant tow forces. Results from the present theory and from the the theory of Gerritsma and Beukelman[5] as computed by Strom-Tejsen et al.[6] are also shown. The two sets of experimental data are in reasonably good agreement, whereas the present theory overpredicts the maximum added resistance by approximately 35% and the theory of Gerritsma and Beukelman overpredicts it by a factor of two.

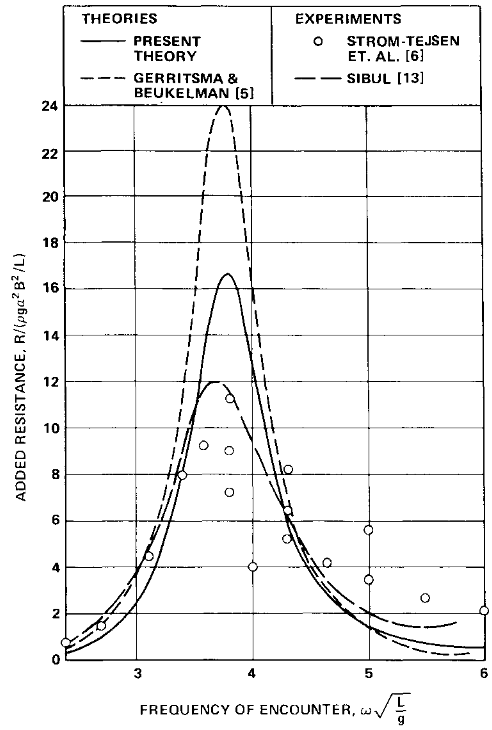

Fig. 6 Comparison of experimental and theoretical added resistance for Series 60 hull with $C_B = 0.60$ at $F_n = 0.283$.

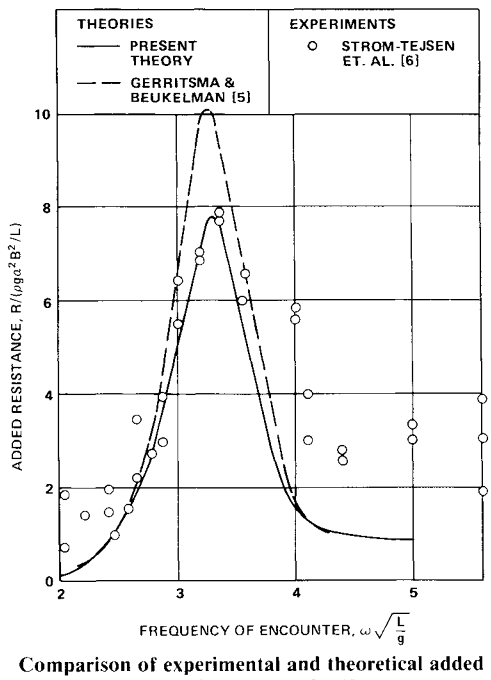

Fig. 7 Comparison of experimental and theoretical added resistance for Series 60 hull with $C_B = 0.80$ at $F_n = 0.165$.

In Fig. 7, the present theory and that of Gerritsma and Beukelman are presented together with experimental data by Strom-Tejsen et al.[6] for the Series 60 hull with $C_B = 0.80$ at $F_n = 0.165$. It is seen that, in the range of the maximum added resistances, the present theory agrees well with the experimental data and that the theory of Gerritsma and Beukelman overpredicts the experimental data by approximately 25%. In the high-frequency (short-wavelength) range, the results of both of the theories are less than half the experimental values. This is probably an indication that for this full hull form the diffraction part of the added resistance, which is the major part at the higher frequencies, may not be accurately predicted by either theory; however, possible

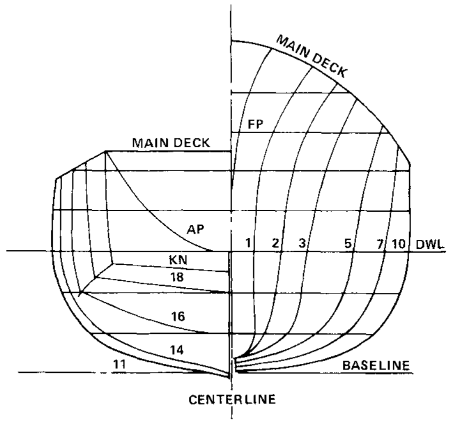

Fig. 8 Body plan of destroyer form.

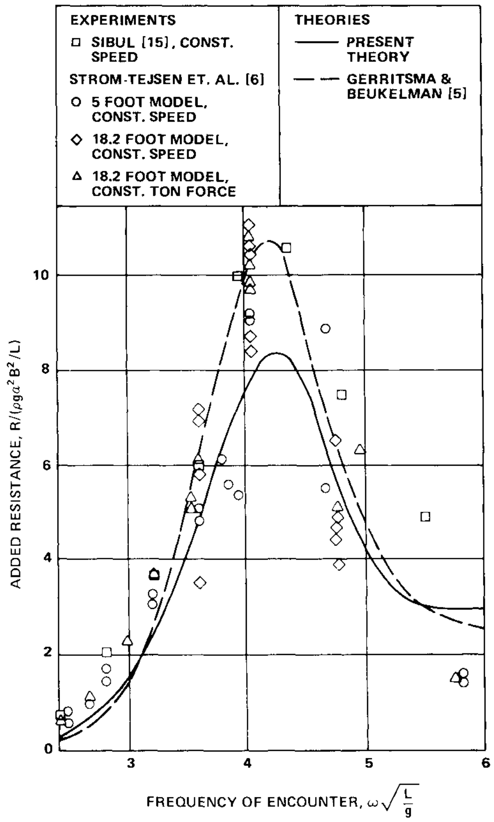

Fig. 9 Comparison of experimental and theoretical added resistance for destroyer hull at $F_n = 0.25$.

inaccuracies in the experiments should also be considered since it is extremely difficult to obtain accurate results in the short-wavelength range.

The body plan of a typical destroyer form for which there exists considerable experimental added-resistance data is given in Fig. 8. This is a high-speed hull form with a transom stern and a very low block coefficient $C_B = 0.49$. Comparisons between the theory of Gerritsma and Beukelman,[5] the present theory, and the experimental data are shown for this destroyer form in Figs. 9 and 10 for $F_n = 0.25$ and $F_n = 0.35$, respectively. For the lower speed case (Fig. 9), data from three sets of experiments conducted at the David W. Taylor Naval Ship Research and Development Center[6] are presented: 5.0 ft model at constant speed and 18.2-ft model at both constant speed and constant tow force, as well as data from constant speed experiments with a 5-ft model conducted at the University of California, Berkeley.[15] The experimental data in Fig. 9 clearly show the large spread that seems to be typical for added resistance data; it is interesting to note that the spread in the experimental data (~30%) is larger than the

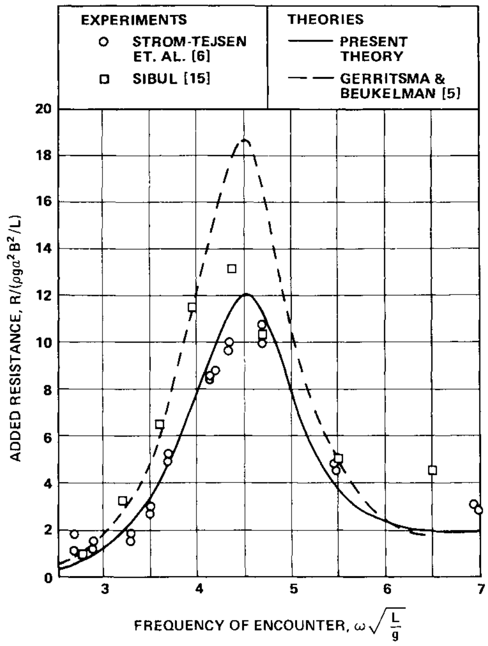

Fig. 10 Comparison of experimental and theoretical added resistance for destroyer hull at $F_n = 0.35$.

difference between the two theories (~25%). For $F_n = 0.25$, the theory of Gerritsma and Beukelman seems to be in closer agreement with the experimental data than does the present theory. On the other hand, for the higher-speed case in Fig. 10 ($F_n = 0.35$), the present theory is closer to the experimental values. Unfortunately, for the higher-speed case, there are only two sets of experimental data; both are from constant-speed tests with 5.0-ft models.

It is important to note from the comparisons in Figs. 9 and 10 that the effect of the increase in the forward speed does not seem to be accurately predicted by these theories for this particular hull form. The experimental data show that there is approximately a 25% increase in the maximum added resistance when the speed is increased from $F_n = 0.25$ to $F_n = 0.35$, whereas the present theory shows an increase in the maximum added resistance of approximately 45% and the theory of Gerritsma and Beukelman shows it to be as much as 70%.

Figure 11 shows the forward-speed-effect comparisons between the present theory and experiments[6] for an additional high-speed hull form that has the body plan shown in Fig. 12. Added-resistance results for four speeds, $F_n = 0.20$, 0.30, 0.40, and 0.50 are presented in Fig. 11. It is seen that the theory predicts almost the same trend in the forward-speed effect on the maximum added resistance as revealed by the experiments; however, it predicts the occurrences of the maximum at higher frequencies than indicated by experimental data. This is quite contrary to what is generally found; namely, there is very good agreement between experiment and theory for the frequency at which the maximum added resistance occurs, as was seen in all the other cases presented in Figs. 5-10. It is difficult to explain the discrepancy between the theory and experiment seen in Fig. 11, but it is encouraging that at least the values of the maximum are predicted accurately for this particular hull form.

VI. Concluding Remarks

The theory presented here expresses the added resistance in oblique waves as the sum of products of first-order terms computed within the computer programs presently in use for predicting heave and pitch motions. Consequently, it is a simple task to prepare a small additional subroutine for computing the added resistance by this theory.

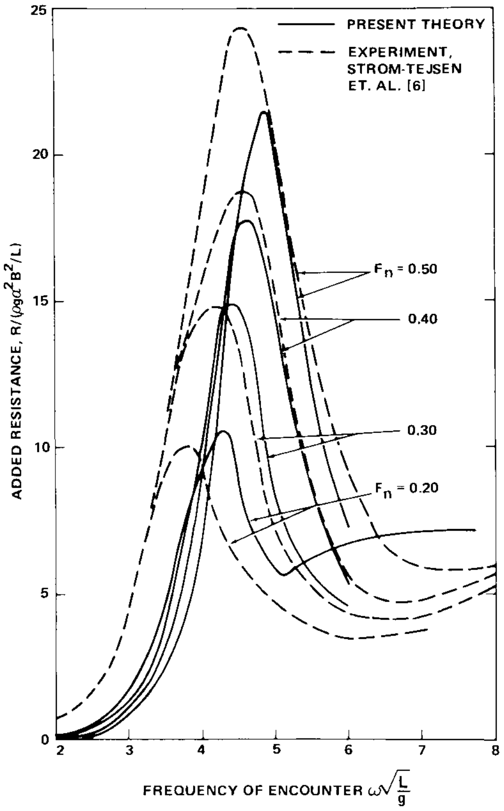

Fig. 11 Comparison of experimental and theoretical added resistance for a high-speed hull form.

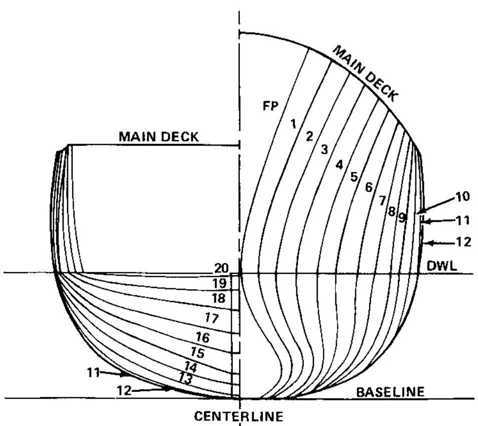

Fig. 12 Body plan of high-speed hull form.

The theoretical results indicate that the maximum added resistance in bow waves ($\beta = 150$ deg) may be at least as large as it is for head waves. For the case of head waves, the present theory predicts added resistances that agree reasonably well with experimental results. However, since the added resistance is a second-order quantity, the agreement between theory and experiment should not be expected to be as good as that usually found for the first-order heave and pitch motions. The differences between the results obtained in different experiments seem to be at least as large as the differences between different theories; a difference of 30% between the experimental values for the maximum added resistances seems to be typical.

It can be concluded that the present theory predicts the added resistance with reasonable accuracy for a wide range of hull forms including hulls with transom sterns and cruiser sterns and with find and full block coefficients over a large speed range. A notable discrepancy between theory and experiment ($\sim 35\%$) was found only for the Series 60 hull with the very fine block coefficient of $C_B = 0.60$. In general, the present theory was found to agree better with experiments than did the theory of Gerritsma and Beukelman.[5]

Appendix

This appendix simplifies the added-resistance term given by Eq. (35) as

$$R_7 = \frac{i}{2} \rho k \cos\beta \int \int_s \phi_7 \frac{\partial \phi_0^*}{\partial n} \, ds \tag{53}$$

to a form that can be computed more easily. The diffraction potential ϕ_7 in Eq. (53) is usually not computed in ship-motion computer programs, since, by use of the Haskin relations, the diffraction part of the exciting force can be expressed in terms of the oscillatory potential ϕ_j. We will show that R_7 can also be expressed in terms of the oscillatory potential by following steps similar to those used in deriving the Haskin relations.

For a slender hull form, the three-dimensional normal derivative may be replaced by the two-dimensional normal derivative as

$$\frac{\partial}{\partial n} \approx \frac{\partial}{\partial N} = N_2 \frac{\partial}{\partial y} + N_3 \frac{\partial}{\partial z} \tag{54}$$

since for a slender ship $n_1 \ll n_2$ or n_3. Here n_1, n_2, and n_3 are the components in the x, y, and z direction of the three-dimensional outward unit normal vector, and N_2 and N_3 are the components in the y and z directions of the two-dimensional normal vector, N in the y-z plane. The normal derivative of the conjugate of the incident-wave potential (9) becomes, with the use of Eq. (54)

$$\frac{\partial \phi_0^*}{\partial n} = \left(iN_2 \sin\beta + N_3 \right) k \phi_0^* \tag{55}$$

Substitution of Eq. (55) into Eq. (53) results in

$$R_7 = \frac{1}{2} \alpha\sigma\rho k \cos\beta \int \int_s \{\exp ik(x\cos\beta + y\sin\beta) + kz\} (iN_2 \sin\beta + N_3)\phi_7 ds \tag{56}$$

The hull condition for the diffraction problem is

$$\frac{\partial \phi_7}{\partial n} = -\frac{\partial \phi_0}{\partial n} \tag{57}$$

which with use of Eq. (54) can be expressed in terms of the two-dimensional sectional normal derivative as

$$\left(N_2 \frac{\partial}{\partial y} + N_3 \frac{\partial}{\partial z} \right) \phi_7 = -\left(N_2 \frac{\partial}{\partial y} + N_3 \frac{\partial}{\partial z} \right) \phi_0 \tag{58}$$

It follows from Eq. (58) that both ϕ_7 and ϕ_0 must have the same x dependence; consequently if we write the incident-wave potential (9) as

$$\phi_0(x,y,z) = e^{-ikx\cos\beta} \psi_0(y,z) \tag{59}$$

we may also write the diffraction potential as

$$\phi_7(x,y,z) = e^{-ikx\cos\beta} \psi_7(y,z) \tag{60}$$

It also follows from Eq. (54) that the hull condition (57) can be expressed as the two-dimensional sectional condition

$$\frac{\partial \psi_7}{\partial N} = -\frac{\partial \psi_0}{\partial N} \tag{61}$$

By substituting Eq. (60) into Eq. (56) we find that

$$R_7 = \tfrac{1}{2}\alpha\sigma k \cos\beta \times \int_s e^{iky\sin\beta + kz}\,(iN_2 \sin\beta + N_3)\,\psi_7 \mathrm{d}s \quad (62)$$

It will be assumed that in Eq. (62) we may replace

$$e^{kz} \text{ by } e^{-ksd} \quad (63)$$

and

$$e^{iky\sin\beta} \text{ by } e^{ik(\pm \frac{1}{2}b)s\sin\beta} \quad (64)$$

where d and b are the sectional drag and beam and s is in sectional-area coefficient. The assumption (63) is often used in strip-theory computations (for example see Ref. 7), and has been shown to give accurate results for conventional ship-hull forms. We have less experience with the second assumption (64), but if the wavelength is considerably larger than the half-beam, it is certainly reasonable. Making the substitutions of Eqs. (63) and (64) into Eq. (62) results in

$$R_7 = \frac{1}{2}\alpha\sigma\rho k\cos\beta \int_L e^{-kds} e^{ik(\pm\frac{1}{2}b)s\sin\beta} \times \int_c \left(iN_2\sin\beta + N_3\right)\psi_7 \mathrm{d}l\mathrm{d}x \quad (65)$$

If we denote the last integral in Eq. (65) by

$$I(x) = \int_c \left(iN_2\sin\beta + N_3\right)\psi_7 \mathrm{d}l \quad (66)$$

and substitute the two-dimensional cylinder-wall condition

$$\frac{\partial\psi_j}{\partial N} = i\omega N_j \text{ for } j=2 \text{ and } 3 \quad (67)$$

into Eq. (66), we find that

$$I(x) = \frac{1}{\omega}\int_c \left(\frac{\partial\psi_2}{\partial N}\sin\beta - i\frac{\partial\psi_3}{\partial N}\right)\psi_7 \mathrm{d}l \quad (68)$$

Green's second identity in two dimensions is

$$\int_c \psi\frac{\partial\varphi}{\partial N}\,\mathrm{d}l = \int_c \varphi\frac{\partial\psi}{\partial N}\,\mathrm{d}l \quad (69)$$

where ψ and φ may be any two-dimensional harmonic function satisfying the free-surface condition

$$\omega^2\psi - g\frac{\partial\psi}{\partial z} = 0 \quad (70)$$

and the condition of outgoing waves at infinity. It should be recalled that the oscillatory potential ϕ_j and the diffraction potential ϕ_7 both satisfy the free-surface condition in the form

$$\left(i\omega - U\frac{\partial}{\partial x}\right)^2\phi + g\frac{\partial}{\partial z}\phi = 0 \quad (71)$$

which reduces to Eq. (70) if the high-frequency assumption, $\omega \gg U(\partial/\partial x)$, used in the ship-motion strip theory, is used. Application of the Green's identity (69) in Eq. (68) leads to

$$I(x) = \frac{1}{\omega}\int_c \left(\psi_2\sin\beta - i\psi_3\right)\frac{\partial\psi_7}{\partial N}\,\mathrm{d}l \quad (72)$$

Use of the cylinder-wall condition (61), substitution of the incident-wave potential (9), and application of the assumptions (63) and (64) allow the integral (72) to be rewritten as

$$I(x) = \alpha\frac{\sigma}{\omega}e^{-kds}e^{-ik(\pm\frac{1}{2}b)s\sin\beta} \int_c \left(\psi_2\sin\beta - i\psi_3\right)\left(N_2\sin\beta + iN_3\right)\mathrm{d}l \quad (73)$$

Substitution of Eq. (73) for the last integral in Eq. (65) results in

$$R_7 = -\frac{1}{2}\alpha^2\frac{\sigma^2}{\omega}\rho k\cos\beta\int_L e^{-2kds}\int_c \times (\psi_3 N_3 + \psi_2 N_2\sin^2\beta)\,\mathrm{d}l\mathrm{d}x \quad (74)$$

where it has been assumed that the sections are symmetric, so that

$$\int_c \psi_3 N_2 \mathrm{d}l = \int_c \psi_2 N_3 \mathrm{d}l = 0 \quad (75)$$

The two-dimensional added-mass and damping coefficients a_{jk} and b_{jk} are usually defined in strip theory (see Eq. 131 in Ref. 7) as

$$a_{jk} - \frac{i}{\omega}b_{jk} = -\rho\frac{i}{\omega}\int_c N_j\psi_k \mathrm{d}l \quad (76)$$

Then substitution of Eq. (76) into Eq. (74) shows that

$$R_7 = -\frac{1}{2}\alpha^2\frac{\sigma^2}{\omega}k\cos\beta\int_L e^{-2kds}(b_{33} + b_{22}\sin^2\beta)\,\mathrm{d}x \quad (77)$$

if only the real part is included.

Acknowledgment

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